XI. Relativity

Today I would like to take you on an incredible journey, this time not far into the universe, but to the world of very fast speeds. I know that this will stretch your imagination to the extreme, and I do not expect that you will retain everything. I hope that a few very basic principles stick though. If you start to have troubles please ask questions! Don't hesitate! Your neighbors may have the same troubles! I will then either try to explain again in a different way and I will point out, what I expect you to accept as facts or where I might have another analogy for you. Many of the things I will tell you are mind boggling, also for me, don't worry! Although I do accept them on the grounds of experiments and mathematical exercises, my mind has still a hard time to visualize the facts.

We ran across the question: what happens when the escape velocity for objects from a heavy compact object, such as a neutron star, increases and increases, and finally approaches the speed of light? Indeed the speed of light is the fastest speed we have heard of. Is it the fastest speed possible? Or can we go beyond? Can we view light as something, which also has to struggle to escape the grip of gravity, like any other object? These are questions, which will lead beyond Newton's view of mechanics, as we shall see. Here we need the help of a young Swiss patent officer, his name: Albert Einstein

Already as a youth he was wondering what was going on. He says:

"There is a paradox upon which I had already hit at the age of 16. If I pursue a beam of light (i.e., he meant a ride on a beam of light) ... I should observe such a beam of light as a spatially oscillating electromagnetic field at rest. However, there seems to be no such thing, whether on the basis of experiments or theory."

What he meant would be the equivalent of riding on an ocean wave, like surfing. Then you see the waves as a stationary wave structure. It is as if a wave were frozen. If you would compare this with your everyday experience of the real world, it looks as if time stands still, when you are traveling along with the beam of light. In his work of a genius Einstein finally concludes that we will never be able to reach this state. A little later we will see why this is indeed impossible and what the important consequences are.

1. Principles of (Special) Relativity:

At first you may think why is this a problem in the first place. The main reason that we think this way is that in everyday life speeds are much slower than the speed of light. Certainly we will run into the question when dealing with impact or escape velocities approaching the speed of light. What if we would try to go beyond this point, can't we just do it? To see the problem we will start with a simple experiment. What is the difference for somebody who moves compared with somebody who stands still? Is there a difference at all?

If I drop a ball while standing still, it just falls perpendicular to the floor, just down.
If I drop a ball, while moving along with the floor, it also hits the floor in the same spot.

For you it looks like the ball is slightly moving in the direction I am walking, but you are outside. You may say: so what? But this is extremely important for the following.

View XI.1: Same experiment on moving railroad car and in the lab.

Assume the experimenter were in a railroad car which is completely closed and moving at a constant speed. He cannot look outside. So he doesn't know whether he is moving or not. He will never find out, not by any experiment he can perform in the car. We can think of many many experiments, they all won't tell whether the car is moving or not. This leads us to the first firm principle:

1) "the laws of physics look the same to any non-accelerating observer"
   (moving with uniform velocity with respect to each other)
If this were not the case, the laws of physics would depend on the state of the individual observer. What a mess!!

Now how do different observers agree about their observations? View XI.2
One of them is moving, the other one is standing outside. Both want to measure velocities. This is a situation, where I find the theory of relativity to be a perfect excuse to play with my son's toys: the Darda cars
As long as the platform remains stationary the observer on the platform and you all see the same velocity. When the platform moves at constant speed, the Darda car still runs at the same velocity as before with respect to the platform, but for you it runs faster here (its speed + the speed of the platform) and slower here (its speed - the speed of the platform).
And now comes the weird thing, which I cannot readily demonstrate in class: How are we doing if both attempt to measure the speed of light? Speed of light +/- speed of the car?

View XI.3: Adding speeds for light.
Surprise, surprise! The speed of light is measured to be exactly the same no matter whether we are moving, how fast we are moving or not at all!! And the theory of electromagnetic waves tells us that it must be indeed so! The non-existence of the change of the speed of light with motion could be verified very accurately, because we all are riding a pretty fast spaceship, our good old Earth. It roams around with about 30 km/sec, and thus the speed of light from stars in different directions would be quite different and different also compared to light from earthly sources.

This leads to the second basic principle (which could be even a part of 1), but it is stated separately, since it is so important):

2) "The speed of light in vacuum is the same for everyone."

What is wrong? Is our math wrong? $300,000 + 100,000 = 300,000$? View XI.4

In physics only what can be measured is real! In order to determine the speed we need to measure Distance and Time. Einstein was so bold to tamper with the common imagination of space and time. He concluded: something must be wrong with our common perception of space and time. Space and time must be different for observers moving at different speed to resolve the problem and to leave physics intact for everybody.

What does it mean to determine a distance and time? To meet somebody (If you set a date, what do you specify? The location where you want to meet and the time when you want to meet) we need to specify 3 coordinates in space and 1 in time. Let's make it simple: we use only 1 coordinate in space (e.g. so many meters from the right edge of the counter)

On my viewgraph space (distance) goes from left to right and time from bottom to top. A point describes a specific event (e.g. the start of a demonstration in this lecture room). How do we describe motion of the light?

A flash is ignited here and it is seen at a distance after it traveled to the location with the speed of light. After 1 year it is seen 1 light year away on both sides. Now let's see what an observer says about this who is moving at half the speed of light View XI.5b

She travels half a light year in 1 year. The observer is in a spaceship and cannot find out that she is moving. For the man on Earth time goes along this line. Both disagree on what time is!!
Time not absolute

Also distance must be described differently, namely such that light has traveled 1 light year to both sides after 1 year. Now the new coordinate system looks skewed, but this is not different from the way we may see a rectangle from the side. I see this page as a perfect rectangle, but to you it looks skewed, whereas the notebook in front of you looks like a perfect rectangle to you, but skewed to me.

In this way Einstein could save both principles at the same time, but at the cost of some weird effects:

- What is simultaneous for the resting observer is not for the moving?

Simultaneity not absolute

Event 1 happens first for observer A, but event 2 happens first for observer B. Now let’s assume, somebody could travel faster than light, say 4 light years in 1 year (4 times the speed of light), then for observer B he would arrive, before he even took off!!! Think what might happen, if you could meet your grandmother, before your father or mother was even born and you shoot her.

Therefore, Einstein concluded that speeds greater than that of light are impossible.

To make the speed of light the same for all observers:

- Time is running slower
- Lengths get contracted

However, both observers can still agree on the combination of space and time. The distance of events in space and time remains unchanged.
This leads to the famous **Traveling twin paradox**. View XI.6, a

Now some of you may ask the tricky question: why is it that the traveler comes back younger and why not the one left on Earth? To the traveler it seemed that the one who remained on Earth was moving (both observers have to see the same!!). The resolution is that only the traveler experienced acceleration, this is what made the difference!

Why can't we go faster than light? The reasoning above seems like a verdict by law not by natural law. As a first answer: the description how to calculate velocities in a moving system tells us immediately that no speed greater than that of light is possible.

And as a nice by-product Einstein found the formula, which is most often cited:

\[ E = mc^2 \]

which means that mass can be readily transformed into energy and vice versa. We can view mass as a "frozen" form of the more violent energy. However, this means whenever an object gains energy, i.e. it becomes faster (it increases its **kinetic energy**), than also its mass must be increased. View XI.7a

As an inevitable consequence, we find:

\[ m \to \text{ as velocity} \to \text{ speed of light} \]

And therefore we can't ever get to the speed of light, since energy goes into increasing mass rather than increasing speed.

Einstein's theory has overthrown almost all views we had before which went back to Newton. However, it did not overthrow Newton's theory as such. Newton's mechanics still works fine for speeds small compared to that of light. Einstein only extended the laws of physics for high speeds.
2. General Relativity:

Now we are back to mass, but we have not touched yet the second aspect of mass, namely gravity. The effects of gravity were not included in Einstein's theory of special relativity. We have only dealt with motion at constant velocity so far. However, we know from prior experience that gravitation causes objects to accelerate, and the strength of gravitation increases proportional to the mass that causes it. The conclusion that mass increases with speed showed Einstein that there must also be a modification in Newton's law of gravity for high speeds. With his success to unify the laws of physics for all observers that with constant velocity, no matter how fast, Einstein tried to include general motion (with changing velocities) as required in gravitational fields.

A) Basic principles

To see how Einstein’s argument goes let's follow someone who has been tricked in a deadly way: View XI.8

The feeling that we float freely tells us that there are no forces around and that we are moving with a constant velocity, right? However, we may be deadly wrong: View XI.8a

View XI.8: Equivalence of floating in space and falling freely.

The same effects would be felt in a defunct elevator, which simply falls freely towards the bottom. The pull of gravitation accelerates the elevator and everything within. So all physical experiments in this elevator box will again give the same answer as the same experiments performed in a spaceship moving at constant velocity through empty space.
In addition, the response to gravity is independent of the object and independent of the object's initial velocity. That this is so can be demonstrated in a nice experiment. This gun is exactly aimed at the monkey. Will I hit or miss it? View XI.9
If we were freely moving at constant velocity with no forces applied, the answer is simple: yes, of course!! View XI.9a
However, if the monkey falls under the force of gravity at the same time? View XI.9b
I trust the principle that all objects respond to gravity in the same way, I'll try:
Both, the monkey and the bullet, fall under the influence of gravity for the same time. Therefore, they meet in the same spot, fatal for the monkey! If you have trouble with the explanation, think of the monkey and the bullet as being both in the elevator box. The situation is equivalent to that in a box, which falls under the influence of a constant acceleration! View XI.9c
As we will see, when we turn the situation around, the equivalence of a system moving at constant velocity and one freely falling in a gravity field is based on
\[ m_i = m_g \]
the inertial mass is equal to the gravity mass of an object.
We have used this since Newton without thinking about it. This is not obvious from the beginning, since gravity and inertia are 2 totally unrelated aspects of mass in Newton's theory. The ability to attract other masses vs. the ability to resist a change in its motion. View XI.10
Now in both cases there is a force which makes the dropped object fall,
in the same way as on Earth. In the second case it is due to the acceleration of the rocket. The dropped object just goes on in the same way as before, since it does not feel the acceleration any more. Here we see the inertia in action. In the example on Earth we see the gravity part of the mass in action. Or putting it on the scales the same weight is shown, independent of whether it is due to the acceleration of the rocket or the gravity on Earth. Therefore, any acceleration is equivalent to gravity, which produces the same acceleration. Watching the astronauts on the Shuttle we can see this effect very lively

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B) Effects of Gravity on Time.

Now let's send light from the bottom of the accelerated rocket to the top. While the light was on the way, the rocket has increased its speed. The observer is moving away from the light source, and we will see a Doppler shift, to be more specific, a red shift. Because gravity is equivalent to acceleration, light from a source in a strong gravity field will also be red-shifted. This effect is called: Gravitational redshift

There are experimental Tests:

White dwarfs have a strong gravitation, since they are so dense. Therefore, spectral lines from white dwarfs are red-shifted!

Laboratory: we can even measure this Doppler shift in the Earth's gravity field with precision instrumentation using the so-called Mößbauer Effect.

A redshift means that the wavelength gets longer and that the frequency of the radiation is reduced. Reducing a frequency has an interesting consequence: we use set frequencies to measure time. For example, the frequency of a pendulum or the frequency of a quartz crystal is used in clocks. All these frequencies must be reduced in a gravity field to be consistent with the behavior of light. This means also that we would see a clock in that gravitation field run slower, if we could observe it. Thus the time runs slower near a huge mass. Here we can construct a Gravitational twin paradox.

Experimental Test:

Atomic clocks (very accurate) were flown on aircraft at different altitudes.
C) Curvature of space.

Let's now look at light, which passes a mass. We can understand the behavior again in comparison with the accelerated rocket. The light will follow a curve, since the rocket is accelerating while the light passes. The same must happen in a gravitational field. This Gravitational deflection of light was observed during a solar eclipse in 1919: It was the first test of one of Einstein's predictions in his general relativity.

The situation looks similar to the wind of interstellar particles focused on the downwind side of the sun by its gravitation. Any object in the universe will do this, the larger the mass the stronger. Galaxies seen behind other galaxies are distorted and deflected.

The effect can even lead to a doubling of the picture. The same quasar, one of the most distant objects in the universe, is seen twice here, since it is behind this huge galaxy which bends the light around.

To get a unique picture for all cases we would like to describe motion in terms of geometry. That was also Einstein's great idea to describe gravitation. How would you define a straight line? Isn't that a line, along which we stare? We cannot stare around a curve, can we? This would be the path of light. But now we have seen it is bent in gravity. We are in trouble with our geometry! - Maybe we define a straight line as the path, which connects 2 points in the shortest possible way. Light is clever, according to a long known principle it always takes the quickest way! Maybe it just takes the quickest way here although it is a curve. This is not so unusual:

*Analogy:* on Earth jet planes will always take the **great circle route** to minimize flight time. The shortest way on a sphere is the great circle. In the geometry on a sphere there is no straight line as there is on a plane.

Obviously mass (and energy) produce a curved space-time. Let's look at a 2 dimensional analogy. This cloth is meant to be the fabric of space-time.

It is flat as long as there are no masses around. The shortest connection between 2 points is a straight line. However, if I throw a metal sphere into it, representing a mass, the cloth is distorted. This is what happens to space in the vicinity of a star. This is impossible to visualize for us (including myself). As the cloth is now bent into the 3rd direction (up down), space is bent into the 4th direction, but where is it? I can't see it, so can't you see me. Therefore, we have to resort to this 2 dimensional analogy. Everything seems to be attracted by this warp in the fabric as in reality. And the shortest pathway is now necessarily curved.
D) Tests of general relativity

Based on this curvature of space and time more effects can be observed which differ from Newton's view of physics.

a) Space time curvature changes motions due to gravity. The Kepler ellipse of the planets is distorted, since it is not drawn on a plane any more!!

*Analogy:* ball rolling on a dented tabletop like the distorted cloth.

-> The orientation of the orbit of Mercury turns around.

Mercury's orbit is not a perfect ellipse ("perihelion shift").

b) Gravitational waves: propagating ripples in space time.

If 2 neutron stars merge or something similar dramatic, a ripple of the distorted space time will be sent out, like light waves, again with the speed of light. They have been predicted by Einstein in analogy to electromagnetic waves, but have not been directly measured yet. They are very weak. *When will they be directly observed?*

So far only indirect evidence has been found: A Pulsar (a very precise clock!) orbiting a second neutron star in a binary system has been observed which is *slowing down* due to *gravity wave emission*. The reduction of the orbital period is exactly as predicted. In 1993 Russell Hulse and Joseph Taylor received the Nobel Prize for this discovery.

E) Black holes.

Now let us move to the final enigma of gravity and relativity! What happens, if the metal ball on the cloth gets heavier and heavier? Finally, the space-time fabric will not withstand any more.

It ruptures and the ball falls into oblivion. This is what a **Black Hole** is about. It constitutes a hole in space time.

*Analogy:* A hole in the fabric of space-time.
The mass falls out of our space, and no information will come back. Why? The matter, which falls into the hole, would reach the speed of light. Light can't come out, since the escape velocity has to exceed the speed of light. Looking at it another way: the gravitational redshift of the waves would be so large that the wavelength goes to infinity, i.e. this is no wave any more.

**a) Principles**

If there is a mass with \( M > 3(?) \ M_{\odot} \) left after a supernova, the collapse of the resulting neutron star continues, since now even the neutron degeneracy pressure can't win any more. Remember anything will break under a certain maximum pressure. And like white dwarfs the neutron star will shrink when it accumulates more mass. Therefore, on the surface **Gravity increases** and beyond a certain point **even light can't escape**.

How do we find something that is not in our space any more and that doesn't emit light? It still makes itself known through its gravity!! If there is an object with more than 3 solar masses in a binary system (where we can verify the mass) and it behaves similarly to a neutron star, i.e. it may accelerate material such that it emits X-rays and gamma-rays, we may have found the fingerprint of a black hole.

**b) Possible example:** Cygnus X-1 is a special binary X-ray source with a total mass of \( \approx 30 \ M_{\odot} \) We see that the normal star moves. From this motion we can conclude that the compact X-ray companion must have \( > 6 \ M_{\odot} \) **evidence** that the compact object must be a black hole.

**c) Features of a Black Hole**

A black hole concentrates mass inside the so-called "Schwarzschild Radius" or the "event horizon". This is the boundary beyond which we cannot see anything. Thus for us no event happens there, since we cannot see it.

Once a black hole has formed it

**Can only grow.** -> Black Holes dig their own holes in space-time.
Can only have mass, angular momentum, electric charge.

It has no shape: "Black holes have no hair."

A black hole of the mass of the sun would have a radius of 3 km.

The radius of a black hole increases with its mass. It is not difficult for super-massive black holes (> $10^8$ solar masses) to form. In fact it is reasonable to assume that in the center of our galaxy as in centers of other galaxies there are huge black holes, as we will see a little later.

If we go to the largest entity our entire universe, we find that the radius of a black hole with the total mass of the universe would be about the size of the universe, as we know it now. Thus is the universe a black hole? Do we live inside a black hole? As we will see, the universe expands rather than collapses, so this must be something different. We will first have to find out more about the structure of the universe and the galaxies, before we talk more about this at the end of the course.

The possibility has been raised that the strong curvature of space time in the vicinity of black holes could lead to a shortcut to other, very distant, places in the universe. Such a potential connection has been coined a 'wormhole'. Based on Einstein’s equations such wormholes have been discussed seriously by scientists, such as Kip Thorne. Although theoretically possible, nobody has come up with a solution to the question, how one could produce such a wormhole and/or keep it open and stable for travelers to pass through.